# NON-LINEAR MASS TRANSFER IN FALLING FILMS

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Abstract—The problem considered is to determine the rate of absorbtion in laminar liquid films for the case of first order chemical reactions taking place in the film. The high reaction rate is the reason for the formation of a very thin concentration boundary layer and hence high concentration gradients. This is equivalent to an additional momentum transfer and the corresponding secondary flow results in non-linear effects in the convective mass transfer. This problem has been solved supposing that the film thickness is much smaller than the length of the film, non-linear effects having been accounted for.

### NOMENCLATURE

- c, molar concentration of the absorbed substance in the film;
- $c_0$ , molar concentration of the absorbent;
- C, dimensionless concentration;
- D, diffusivity;
- Da, Dammköhler number;
- Fo, Fourier number;
- *Fr*, Froude number;
- g, gravitational constant;
- h, film thickness;
- $h_0$ , initial film thickness;
- *H*, dimensionless film thickness;
- *I*, total flux of the absorbed substance through the interface;
- j, diffusion flux of the absorbed substance; J, rate of mass transfer;
- k, rate constant of chemical reaction;
- *M*, molecular mass of the absorbed substance;
- $\mathcal{M}_0$ , molecular mass of the absorbent;
- **n**, unit vector normal to the film surface;
- p, pressure;
- $p_0$ , initial pressure;
- P, dimensionless pressure;
- $Q_0$ , initial liquid flow rate of the film;
- Re, Reynolds number;
- *u*, *x*-component of the velocity;
- $u_0$ , initial velocity;
- $U_0$ , characteristic velocity;
- $u_x$ , velocity of the infinitely long film;
- U, X-component of the dimensionless velocity;
- v, y-component of the velocity;
- V, Y-component of the dimensionless velocity;
- v, statistically averaged velocity vector of the substance transferred;
- v<sub>0</sub>, barocentric velocity vector of the pure absorbent;
- v<sub>1</sub>, barocentric velocity vector of the liquid (solvent included);
- We, Weber number;
- x, longitudinal coordinate;
- X, dimensionless longitudinal coordinate;

- *y*, transverse coordinate;
- Y, dimensionless transverse coordinate.

Greek symbols

- $\alpha$ , relative density difference;
- $\beta$ , mass transfer coefficient accounting for non-linear effects;
- $\beta_0$ , mass transfer coefficient, not accounting for non-linear effects;
- $\varepsilon$ ,  $\varepsilon_0$ , small parameters;
- $\mu$ , dynamic viscosity;
- v, kinematic viscosity;
- $\delta$ , concentration boundary layer thickness;
- $\rho$ , density;
- $\theta$ ,  $\theta_0$ , small parameters;
- $\eta$ , dimensionless variable.

# Superscripts

\*, quantities at the film surface;

, ", ordinary functions derivatives.

# **I. INTRODUCTION**

ONE OF the basic methods for intensification of the industrial absorption processes is the creating of high concentration gradients. Usually, this is realized by means of absorbents, that include a component which reacts chemically with the absorbed gas. Thus the thickness of the concentration boundary layer in the liquid phase is strongly reduced and a high concentration gradient is obtained which controls interphase mass transfer. So the particles which are exchanged between the phases (transfer linear momentum) induce additional motion inside the phase which is perpendicular to the interface and co-linear to the mass transfer direction. As a result the mass transfer in the liquid phase influences its flow; the velocity distribution depends upon the concentration of the absorbed substance. Thus the convective diffusion equation is no longer linear. In what follows this effect will be referred to as a non-linear effect of the first kind.

The considerable variations of the concentration of the absorbed substance in the liquid phase induces changes of the parameters of the absorbent, in particular its density, viscosity and diffusivity, which depend upon its concentration. In this way additional nonlinear effects arise in hydrodynamics and mass transfer in the liquid phase; these will be referred to as nonlinear effects of the second kind.

The theory of transfer processes in systems of intensive mass transfer has been subject to numerous investigations for the case of mass transfer in solid-liquid systems [1]. They are associated with the intensification of industrial electrochemical processes and the solution of problems in the field of electrochemical formation and the accuracy of electrochemical machining [2].

Absorption in the presence of high concentration gradients has not been studied very intensively [3]. Thus, the theoretical investigation of the kinetics of the mass transfer in the liquid phase for the case of absorbtion of gases in laminar liquid films in the presence of fast chemical reaction in the liquid is of considerable interest. This will be the subject of the present paper.

#### 2. CONVECTIVE MASS TRANSFER IN A FALLING FILM IN THE PRESENCE OF HIGH CONCENTRATION GRADIENTS

The mathematical model of convective mass transfer in a falling film comprises a system of five partial differential equations for the components of the velocity, the pressure, the concentration and the thickness of the film:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \\
= \rho g - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\frac{2}{3}\mu\left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right] \\
+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right],$$
(1)
$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x}\right)$$

$$\begin{pmatrix} \partial x & \partial y \end{pmatrix} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ \frac{2}{3} \mu \left( 2 \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \right) \right],$$

$$(2)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0, \qquad (3)$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial c}{\partial y}\right) - kc, \qquad (4)$$

$$Q_0 + \int_0^x I \, dx = \int_0^h \rho u \, dy, \tag{5}$$

defined in a coordinate system, where the x-axis coincides with the wall and the y-axis is directed into the film.

In equations (1) and (5) the density, viscosity and diffusivity depend upon the concentration of the diffusing substance. This is why three more equations are necessary where the latter can be represented by means of their values for the pure absorbent,  $\rho_0$ ,  $\mu_0$ ,  $D_0$ , multiplied by correction factors  $\rho_1$ ,  $\mu_1$ ,  $D_1$ , depending upon the coordinates:

$$\rho = \rho_0 \rho_1, \tag{6}$$

$$\mu = \mu_0 \mu_1, \tag{7}$$

$$D = D_0 D_1. \tag{8}$$

The boundary conditions for equation (1) and (5), have the form

$$x = 0, u = u_0, v = 0, p = p_0, h = h_0, c = 0;$$
 (9)

$$x \to \infty, u = u_x(y), v = 0, c = c^*;$$
 (10)

$$y = 0, u = 0, v = 0, \frac{\partial c}{\partial y} = 0; \qquad (11)$$

$$y = h, c = c^*;$$
 (12)

$$p^* + 2\mu \frac{1 + h'^2}{1 - h'^2} \left(\frac{\partial u}{\partial x}\right)_{y=h} + \frac{\sigma h''}{(1 + h'^2)^{3/2}} = 0; \quad (13)$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=h} + \left(\frac{\partial v}{\partial x}\right)_{y=h} - \frac{4h'}{1-h'^2} \left(\frac{\partial u}{\partial x}\right)_{y=h} = 0; (14)$$

$$\left(\frac{\partial c}{\partial y}\right)_{y=h} - h' \left(\frac{\partial c}{\partial x}\right)_{y=h} = \frac{\rho_0}{\mathscr{M}D} (h'u^* - v^*). \quad (15)$$

Equations (13) and (14) express the absence of hydrodynamic influence on the gas phase<sup>†</sup>. High concentration gradients influence on the hydrodynamics is accounted for by means of equation (15), the derivation of which is presented in what follows.

The diffusion flux of the transferred substance in each space position of the liquid film can be expressed [1] through the statistically averaged velocity of the particles (molecules, atoms, ions) motion of the transferred substance v and the barocentric velocity  $v_1$  of the liquid mixture as a whole:

$$\mathbf{j} = \mathcal{M}c(\mathbf{v} - \mathbf{v}_1). \tag{16}$$

 $\mathbf{v}_1$  satisifies the equations of hydrodynamics and is connected with the velocities of the absorbent's components:

$$\rho \mathbf{v}_1 = \mathcal{M}_0 c_0 \mathbf{v}_0 + \mathcal{M} c \mathbf{v}, \tag{17}$$

where  $\rho$  is the total density of the absorbent:

$$\rho = \mathcal{M}_0 c_0 + \mathcal{M} c = \rho_0 + \mathcal{M} c. \tag{18}$$

The projection of the vector equation (17) along the normal of the film's surface for y = h yields:

$$\rho^*(\mathbf{v}_1^*, \mathbf{n}) = \mathscr{M}c^*(\mathbf{v}^*, \mathbf{n})$$
(19)

and having in mind that the film's surface is impermeable for the molecules of the absorbent

$$(\mathbf{v}_0^*, \mathbf{n}) = 0.$$
 (20)

<sup>†</sup>It is proposed that the rate of absorption is limited by the liquid phase mass transfer.

In equations (19) and (20) and in what follows the expressions in brackets define the scalar product of two vectors. From equation (19) it follows that the flow of the absorbed substance through the interface has both diffusive and convective components:

$$I = - \mathcal{M}D\left(\frac{\partial c}{\partial n}\right)_{y=h} + \mathcal{M}c^{*}(\mathbf{v}_{1}^{*}, \mathbf{n}), \qquad (21)$$

where  $\partial/\partial n$  is the derivative along the normal of the film's surface.

The diffusive component in equation (21) can be expressed in terms of the projection of equation (16) along the normal of the film's surface at y = h:

$$(\mathbf{j^*}, \mathbf{n}) = -\mathcal{M}D\left(\frac{\partial c}{\partial n}\right)_{y=h} = \mathcal{M}c^*(\mathbf{v^*}, \mathbf{n}) - \mathcal{M}c^*(\mathbf{v_1^*}, \mathbf{n})$$
(22)

Equations (19), (21) and (22) yield:

$$I = \rho^* \frac{-h'u^* + v^*}{\sqrt{(1+h'^2)}} = \frac{\mathcal{M}D\rho^*}{\rho_0}$$
$$\times \frac{h'\left(\frac{\partial c}{\partial x}\right)_{y=h}}{\sqrt{(1+h'^2)}} - \left(\frac{\partial c}{\partial y}\right)_{y=h}}, \quad (23)$$

whence equation (15) follows directly.

The simultaneous solution of equations (13) and (15) enables the mass transfer rate in the liquid film to be determined:

$$J = \beta \mathscr{M} c^* = -\frac{1}{l} \int_0^l I \, \mathrm{d}x, \qquad (24)$$

where the mass transfer coefficient is defined as

$$\beta = \frac{1}{l \mathcal{M} c^*} \int_0^l \rho^* \frac{h' u^* - v^*}{\sqrt{(1 + h'^2)}} \, \mathrm{d}x.$$
 (25)

The order of the separate effects, present in equations (1) and (15), can be found after introducing the following dimensionless quantities:

$$x = lX, \quad y = h_0 Y, \quad u = u_0 U, \quad v = \varepsilon u_0 V,$$

$$p = \rho_0 u_0^2 P, \quad h = h_0 H, \quad c = c^* C, \quad \varepsilon = h_0 / l,$$

$$Re = \frac{u_0 h_0 \rho_0}{\mu_0}, \quad Fr = \frac{u_0^2}{gh_0}, \quad We = \frac{\rho_0 h_0 u_0^2}{\sigma},$$

$$D_0 l \qquad kl \qquad \mathcal{M}c^* \quad o^* - \rho_0$$
(26)

$$Fo = \frac{D_0 l}{u_0 h_0^2}, \quad Da = \frac{kl}{u_0} \qquad \alpha = \frac{\mathcal{M}c^*}{\rho_0} = \frac{\rho^* - \rho_0}{\rho_0}.$$

Thus, the system (1) and (15) takes the form:

$$\epsilon \rho_{1} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = \frac{\rho_{1}}{Fr} - \epsilon \frac{\partial P}{\partial X} + \frac{1}{Re} \left\{ \epsilon^{2} \frac{\partial}{\partial X} \left[ \frac{2}{3} \mu_{1} \left( 2 \frac{\partial U}{\partial X} - \frac{\partial V}{\partial Y} \right) \right] + \frac{\partial}{\partial Y} \left[ \mu_{1} \left( \frac{\partial U}{\partial Y} + \epsilon^{2} \frac{\partial V}{\partial X} \right) \right] \right\},$$
(27)
$$\epsilon^{2} \rho_{1} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = - \frac{\partial P}{\partial Y}$$

$$+ \frac{\varepsilon}{Re} \left\{ \frac{\partial}{\partial X} \left[ \mu_1 \left( \frac{\partial U}{\partial Y} + \varepsilon^2 \frac{\partial V}{\partial X} \right) \right] \right. \\ \left. + \frac{\partial}{\partial Y} \left[ \frac{2}{3} \mu_1 \left( 2 \frac{\partial V}{\partial Y} - \frac{\partial U}{\partial X} \right) \right] \right\},$$
(28)

$$\frac{\partial}{\partial X}(\rho_1 U) + \frac{\partial}{\partial Y}(\rho_1 V) = 0, \qquad (29)$$

$$U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = Fo\left[\varepsilon^{2}\frac{\partial}{\partial X}\left(D_{1}\frac{\partial C}{\partial X}\right) + \frac{\partial}{\partial Y}\left(D_{1}\frac{\partial C}{\partial Y}\right)\right] - DaC, \quad (30)$$

$$1 + \int_0^X \frac{\rho^*}{\rho_0} \frac{V^* - H'U^*}{\sqrt{(1 + \varepsilon^2 H'^2)}} dX = \int_0^H \rho_1 U \, dY; (31)$$

$$X = 0, U = 1, V = 0, P = p_0/\rho_0 u_0^2, H = 1, C = 0;$$
(32)

$$\infty, U = u_x(h_0 Y)/u_0, V = 0, C = 1;$$
 (33)

$$Y = 0, U = 0, V = 0, \frac{\partial C}{\partial Y} = 0;$$
 (34)

$$Y = H, C = 1;$$
 (35)

$$P(X, H) + \frac{2\varepsilon\mu_1}{Re} \frac{1+\varepsilon^2 H'^2}{1-\varepsilon^2 H'^2} \left(\frac{\partial U}{\partial X}\right)_{Y=H}$$

 $X \rightarrow$ 

$$+\frac{\varepsilon^2 H''}{We(1+\varepsilon^2 H'^2)^{3/2}}=0; \quad (36)$$

$$\left(\frac{\partial U}{\partial Y}\right)_{Y=H} + \varepsilon^2 \left(\frac{\partial V}{\partial X}\right)_{Y=H} - 4\varepsilon^2 \frac{H'}{1 - \varepsilon^2 H'^2} \left(\frac{\partial U}{\partial X}\right)_{Y=H} = 0;$$
(37)

$$\varepsilon^{2}H'\left(\frac{\partial C}{\partial X}\right)_{Y=H} - \left(\frac{\partial C}{\partial Y}\right)_{Y=H} = \frac{1}{\alpha D_{1}Fo}(1'^{*} - H'U^{*}). \quad (38)$$

From equations (27) and (38) it can be concluded that high concentration gradients cause non-linear effects of the first and second kind. The first kind effect is determined by the additional flow, accounted by equation (38). The second kind effects arise because of the variable physico-chemical properties of liquid of the film.

# 3. APPROXIMATIONS FOR THE FILM FLOW

For most practical applications of absorbtion in a falling film, the length of the film is much larger than its thickness

$$\varepsilon \ll 1.$$
 (39)

This fact enables equations (27) and (38) to be solved for the zeroth approximation with respect to  $\varepsilon$ .

For cases when high concentration gradients are generated as a consequence of a volumetric chemical reaction in the liquid of the film the second kind nonlinear effects can be neglected by comparison with the first kind ones. The reason for this is the fact that the concentration gradients are high mostly because the concentration boundary layer is very thin. That is, the transverse changes of the concentration in the concentration boundary layer do not require its accounting for in the physico-chemical properties of the absorbent. From equations (6), (18) and (26) it is seen that  $\rho_1 = 1 + \alpha C$ . From practical considerations  $\rho_1 = 1$  as far as usually  $\alpha \leq 10^{-2}$  even at high pressures. Compared to the specific density, the viscosity and diffusivity depend more weakly upon the concentration and therefore  $\mu_1 = 1$ ,  $D_1 = 1$ .

Keeping in mind the considerations mentioned above, the problem, equations (1) and (15); equations (27) and (38), respectively; will be solved for the zeroth approximation in  $\varepsilon$ ; that is, accounting for the primary effects due to the high concentration gradients. Thus, equations (1) and (15) lead to three coupled boundary value problems for the components of the velocity, the concentration of the absorbed gas and the film thickness:

$$v \frac{\partial^2 u}{\partial y^2} + g = 0, \ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0;$$

$$y = 0, u = v = 0; y = h(x), \frac{\partial u}{\partial y} = 0;$$
 (40)

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - kc;$$
  

$$x = 0, c = 0; y = 0, \frac{\partial c}{\partial y} = 0; y = h, c = c^*;$$
(41)

$$\begin{pmatrix} \frac{\partial c}{\partial y} \end{pmatrix}_{y=h} = \frac{\rho_0}{MD} (h'u^* - v^*);$$

$$x = 0, \ h = h_0.$$

$$(42)$$

The solution of equation (40) can be obtained directly with an accuracy of an arbitrary function h(x):

$$u = \frac{g}{2v} (2hy - y^2), v = -\frac{g}{2v} h' y^2.$$
 (43)

Next, it is necessary to obtain the solution of equation (41) with the same accuracy h(x). Introducing the solutions of equations (40), (41) into equation (42) yields a condition for the determination of h(x). To solve equation (41) it is necessary to fix the characteristic quantities of the process, the most important of which is the thickness of the diffusion boundary layer, which can be estimated from:

$$D\frac{\partial^2 c}{\partial y^2} \sim kc, \ c \sim c^*, \ y \sim \delta, \tag{44}$$

whence

$$\delta = \sqrt{\frac{D}{k}}.$$
 (45)

Estimation of the separate effects in equation (41) is possible after introducing the following dimensionless variables and parameters

$$x = lX, y = h - \delta\eta, c = c^*C, h = h_0H,$$
  

$$\theta = \frac{\delta}{h_0} = \frac{1}{h_0} \sqrt{\frac{D}{k}}, \ \theta_0 = \frac{U_0h_0}{l\sqrt{kD}}, \ U_0 = \frac{gh_0^2}{2\nu}.$$
(46)

In this way equation (41) take the form:

$$\theta\theta_{0}(H^{2} - \theta^{2}\eta^{2})\frac{\partial C}{\partial X} + \theta_{0}(2H^{2}H' - 2\theta H H'\eta)$$
$$\times \frac{\partial C}{\partial \eta} = \frac{\partial^{2}C}{\partial \eta^{2}} - C; \quad (47)$$

 $X = 0, C = 0; \eta = 0, C = 1; \eta \to \infty, C = 0.$ 

Introducing equation (46) into equation (42) yields

$$-\varepsilon_0 \left(\frac{\partial C}{\partial \eta}\right)_{\eta=0} = 2H^2 H', \qquad (48)$$

therefore mass transfer will influence the hydrodynamics when

$$\varepsilon_0 = \frac{\alpha \sqrt{kD}}{\varepsilon U_0} > 10^{-2} \tag{49}$$

or

$$\alpha \sqrt{k} > 1, \tag{50}$$

i.e. when  $k > 10^4 s^{-1}$  and  $\alpha < 10^{-2}$ .

### 4. MASS TRANSFER IN THE PRESENCE OF FAST CHEMICAL REACTION

For the cases when  $k > 10^4 s^{-1}$  it can be shown that  $\theta < 10^{-2}$  which means that the longitudinal convective transfer of substance is negligible compared to the transverse one and hence the boundary value problem equation (47) can be simplified to the zeroth approximation in  $\theta$ :

$$2\theta_0 H^2 H' \frac{\partial C}{\partial \eta} = \frac{\partial^2 C}{\partial \eta^2} - C;$$
  
(51)  
$$\eta = 0, C = 1; \eta \to \infty, C = 0.$$

The solution of equation (51) is straightforward:

$$c = c^* \exp\left\{-\left[\sqrt{\left(\frac{U_0^2 h^4 h'^2}{D^2 h_0^4} + \frac{k}{D}\right) - \frac{U_0 h^2 h'}{D h_0^2}}\right] \times (h - y)\right\}.$$
 (52)

Introducing equation (52) in equation (42) yields for film thickness:

$$h = h_0 \sqrt[3]{\left(1 + \frac{3}{2}\varepsilon_0 X \sqrt{\frac{\rho_0}{\rho^*}}\right)}$$
(53)

Introducing equations (43) and (53) into equation (25) enables one to find the mass transfer coefficient:

$$\beta = \sqrt{\left(kD\frac{\rho^*}{\rho_0}\right)}.$$
 (54)

For the case of a fast chemical reaction, neglecting non-linear effects it was obtained [4]:

$$\beta_0 = \sqrt{(kD)}.$$
 (55)

Equations (54) and (55) render the interesting result :

$$\frac{\beta}{\beta_0} = \sqrt{\frac{\rho^*}{\rho_0}},\tag{56}$$

which means that the first kind non-linear effects depends upon the thermodynamic equilibrium only.

### 5. MASS TRANSFER FOR VERY FAST CHEMICAL REACTIONS

For very large values of  $k(k > 10^8 s^{-1})$  the parameter  $\theta_0$  in equation (47) becomes small ( $\theta_0 < 10^{-2}$ ), that is, the diffusion boundary layer thickness is very small and the molecular transfer is predominant. In this case the boundary value problem can be written for the zeroth approximation in  $\theta_0$ :

$$\frac{\partial^2 C}{\partial \eta^2} - C = 0; \tag{57}$$

$$\eta = 0, C = 1; \eta \to \mathcal{X}, C = 0.$$

The solution of equation (57) is straightforward:

$$c = c^* \exp\left[(y-h)\sqrt{\frac{k}{D}}\right]$$
(58)

and it enables the thickness of the film and the mass transfer coefficient to be found:

$$h = h_0 \sqrt[3]{\left(1 + \frac{3}{2}c_0 X\right)},$$
 (59)

$$\beta = \frac{\rho^*}{\rho_0} \sqrt{(kD)} \tag{60}$$

which means that the non-linear effects depend again upon the thermodynamic equilibrium only, but the dependence is stronger:

$$\frac{\beta}{\beta_0} = \frac{\rho^*}{\rho_0}.$$
 (61)

# 6. CONCLUSION

The analysis of the results obtained for the absorbtion kinetics of falling liquid films accompanied by an irreversible chemical reaction of the first order yields the following important conclusions:

(a) In the presence of high rate chemical reactions in falling films non-linear mass transfer takes place.

(b) Non-linear effects result in considerable modifications in the film flow.

(c) Mass transfer kinetics is influenced weakly by the non-linear effects because in the presence of fast chemical reactions the mass transfer coefficients do not depend upon hydrodynamics. The weak effects are the result of the modified interface.

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## TRANSFER DE MASSES NON-LINEAIRE DANS UN FILM LIQUIDE COULANT

Résumé—On a discuté le problème de la détermination de la vitesse d'absorption dans un film liquide laminaire dans les cas de réaction chimique de premier ordre dans le volume du film. La grande vitesse de la réaction est cause de l'amincissement de la couche frontière ce qui mène à la formation d'un grand gradient de concentration. Cela est équivalent au transfer d'impulse et le courant additional mène à des effects dans le transfer de masses convectif. Le problème est résolu en admet tant que l'épaisseur du film est beaucoup plus petite que sa longueur et en rendant compte des effects non-linéaires.

## NICHTLINEARE MASSENÜBERTRAGUNG IN EINER FLIESSENDEN FLÜSSIGKEITSSCHICHT

Zusammenfassung—Es wird die Aufgabe zur Bestimmung der absorptionsgeschwindigkeit in einer laminaren Flüssigkeitsschicht unter Auftraten einer chemischen Reaktion erster Ordnung betrachtet. Die grosse Reaktionsgeschwindigkeit führt zur Verringerung der Stärke der Diffusionsgrenzschicht, was die Bildung eines hohen Konzentrationsgradienten bedingt. Dieses ist äquivalent der Impulsübertragung. Die zusätzliche Strömung führt zu nichtlinearen Effekten in der konvektiven Massenübertragung.

Die Aufgabe wird mit der Bedingung gelöst, dass die Stärke der Flüssigkeitsschicht um vieles geringer ist als ihre Länge und unter Berücksichtigung dieser nichtlinearen Effekte.

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# НЕЛИНЕЙНЫЙ МАССОПЕРЕНОС В СТЕКАЮЩИХ ПЛЕНКАХ

Аннотация — Решена задача о скорости абсорбции ламинарной пленкой при протекании в жидкости химической реакции первого порядка. Из-за большой скорости реакции и соответствующего утоньшения диффузионного пограничного слоя возникает большой градиент концентрации. Это эквивалентно дополнительному переносу импульса, появлению вторичных течений и нелинейных эффектов в конвективном массопереносе. Задача решается с учетом нелинейных эффектов в предположении, что толщина пленки намного меньше ее длины.